The Heat Capacity of ^{4}He under Rotation Near T_λ. *

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The heat capacity of ^{4}He in a small rotating cylinder near the lambda transition is computed using the Ψ theory of Ginzburg and Sobyanin. We obtained a phase diagram for vortex formation which agrees with the computation of Kiknadze and Mamaladze.

1. INTRODUCTION

The Lambda- transition in liquid helium has become a model for phase transitions. The transition with no counterflow between normal fluid and superfluid is rather well studied and understood in terms of renormalization group theory.

Broken gauge symmetry gives rise to a new thermodynamic degree of freedom below the superfluid transition. It corresponds to counterflow of normal and superfluid Colll])oll ('ills." It is analogous to magnetic transitions in antiferromagnets under applied magnetic field. The transition under the counterflow has been studied less thoroughly.

One can study transition either keeping the counterflow current or counterflow velocity constant. Only the first situation has been examined experimentally [1]. Most of the theories deal with the transition at constant velocity in one-dimensional geometry [2], [3]. However, it is easier to realize the constant velocity case in a rotating container. The normal fluid follows the rotation of the container and the superfluid is either stationary or forms quantized vortices. Thus the counterflow state is well defined as long as vortex pattern is known.

2. APROACH

We consider here a situation analogous to the experiment planned at JPL. The helium is confined to an array of thin capillaries (of which we consider one) with their axes parallel to the axis of rotation.

The phase diagram of this system in the R plane has been calculated in [4], using the same approach as ours.

We use the stationary equations of the phenomenological theory of superfluidity (Ψ theory [3]). Although less rigorous than a proper renormalization-group treatment, this theory should provide starting point for comparison with future experimental results.

We used the Ψ theory free energy expansion to find the distribution of the order parameter that minimizes the free energy of helium in the cell. The solution is then substituted into the expression for free energy. The heat capacity is calculated from the integrated free energy by taking the second derivative of the free energy as a function of temperature.

We only consider the cases of no vortices ($N = 0$) or one vortex in the cylinder ($N = 1$). In these cases the cylindrical symmetry will be preserved.

In cylindrical coordinates the free energy of helium in the rotating vessel is:

$$\Phi = \tau^2 T_\lambda \Delta C \left(\left(\frac{df}{d\tilde{r}} \right)^2 + \left(\frac{N^2}{\tilde{r}^2} + \tilde{\omega}^2 \tilde{r}^2 - 2\tilde{\omega} N + \frac{3}{3+M} \right) f^2 + \frac{3}{3+M} \left(-\frac{1+M}{2} f^4 + \frac{M}{3} f^6 \right) \right)$$

here $\tau = (T - T_\lambda)/T_\lambda$, $\tilde{r} = r/\xi(\tau)$, $\tilde{\omega} =$., ()//711 $/\xi^2(\tau))$, f is the magnitude of the order parameter, normalized by its bulk value with zero counterflow, r is distance from the axis of the cylinder, ω is rotation velocity, $\xi(\tau)$ is the temperature-dependent coherence length and M is an unknown parameter of the theory that has to be measured experimentally.

Assuming the axisymmetric case, minimization of this energy over the cylinder leads to the following differential equation for the magnitude of the order parameter:

* Supported by NASA

$$\frac{d^2 f}{d\tilde{r}^2} + \frac{1}{\tilde{r}}\frac{df}{d\tilde{r}} - f\left(\tilde{\omega}^2\tilde{r}^2 - \frac{3}{3+M} - 2\tilde{\omega}N + \frac{N^2}{\tilde{r}^2}\right) +$$
$$+ \frac{3}{3+M}\left(\frac{1-M}{2}f^3 + \frac{M}{3}f^5\right) = 0$$

We solve this equation numerically using asymptotical solution $f \sim C\tilde{r}^N$ at zero. The parameter C is adjusted to satisfy the boundary condition $f = 0$ at the wall.

3. RESULTS

We calculated the heat capacity for $R = 30\mu m$, $\omega = 0, 50, 100, 150, 200$ rad/sec, $N = 0, 1$ and $M = 0, 1$. The results are shown in the figure The plots from top to bottom correspond to $N = 0, M = 1$, $N = 1, M = 0$, $N = 0, M = 1$ and $N = 1, M = 1$. On each plot the lines from top (solid line) to bottom correspond to increasing rotation velocity. The left end of each curve is the end of the stability region of the superfluid phase. The transition temperature is suppressed by both size effect and counterflow. The horizontal dotted line on each plot shows the bulk heat capacity in the absence of counterflow.

At lower rotation velocities $w = 0, 50$ rad/sec the $N = 0$ state is thermodynically stable. At higher rotation velocity only the $N = 1$ state is stable. At $w = 100, 150$ rad/sec a first order transition $N = 0 \to N = 1$ takes place as the temperature is reduced further from T_λ.

At $M = 1$ the heat capacity does not diverge at the transition, although the increase is much more prominent than for $M = 0$. It should be pointed out that for $M > 1$ the transition in this geometry becomes first-order [4].

Our results can be re-scaled for different values of capillary sizes (R), rotation velocities and temperatures as $R \to kR$, $w \to \omega/k^2$, $\tau \to k^{-2/\nu}\tau$.

Finally we have calculated the latent heat of the $N = 0 \to N = 1$ transition. The values are

w	M	$T_\lambda - T$, K	ΔQ, joule/m^3
100	0	2.50 .10-7	0.0072
150	0	2.4 .10-7	0.0012
100	1	2.84 · 10-7	0.0126
150	1	3.0 · 10-7	0.0093

4. CONCLUSIONS

We have calculated the heat capacity of superfluid ^{4}He contained in a rotating capillary based on the Ψ theory expansion of the free energy. Our calculations show that the effect of counterflow on heat capacity can be measured with realistic values of experimental parameters.

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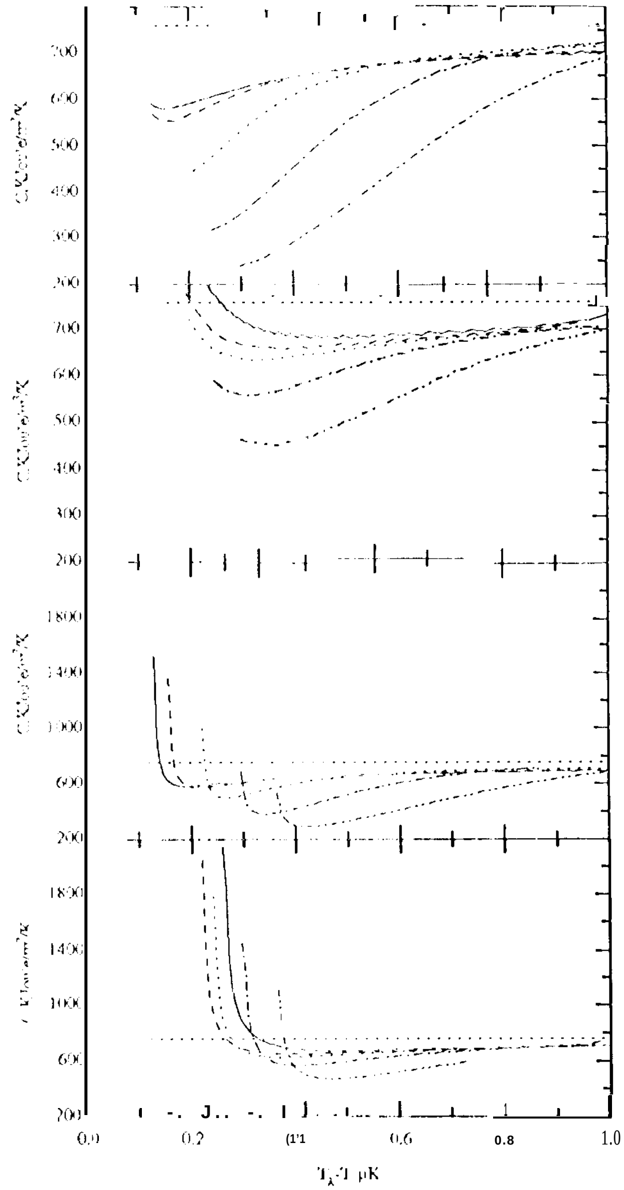

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